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AVERAGE AND PROBABILITY.

89. **Proposed by B. F. FINKEL, A.M., M.Sc., Professor of Mathematics and Physics, Drury College, Springfield, Mo.**

An inch auger-hole is bored at random through a six-inch sphere. Find the average volume of the auger-hole.

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.

Let $x^2+y^2+z^2=R^2$ be the equation to the sphere.

Let $(x-a)^2+y^2=r^2$ be the equation to the auger.

Also let $\sqrt{[r^2-(x-a)^2]}=m$, $\sqrt{[R^2-r^2+a^2-2ax]}=n$.

From $a=0$, to $a=R-r$,

$$V=4\int_{a-r}^{a+r}\int_0^m\sqrt{[R^2-x^2-y^2]}dxdy=2\int_{a-r}^{a+r}mndx$$

$$+2\int_{a-r}^{a+r}(R^2-x^2)\sin^{-1}\left(\frac{m}{\sqrt{(R^2-x^2)}}\right)dx=2\int_{a-r}^{a+r}mndx$$

$$-\tfrac{2}{3}\int_{a-r}^{a+r}\frac{(3R^2x-x^3)(aR^2-R^2x+r^2x-a^2x+ax^2)dx}{mn(R^2-x^2)}$$

$$=2\int_{a-r}^{a+r}mndx+\tfrac{2}{3}\int_{a-r}^{a+r}\frac{nx^2dx}{m}-\tfrac{2}{3}\int_{a-r}^{a+r}\frac{x(R^2-x^2)dx}{mn}+\tfrac{4}{3}R^2\int_{a-r}^{a+r}\frac{mdx}{n}$$

$$-\tfrac{4}{3}R^2\int_{a-r}^{a+r}\frac{x(a-x)dx}{mn}-\tfrac{2}{3}R^3\int_{a-r}^{a+r}\frac{mdx}{(R+x)n}-\tfrac{2}{3}R^3\int_{a-r}^{a+r}\frac{mdx}{(R-x)n}.$$

Let $x=a-r\cos2\theta$, $R^2-(a-r)^2=b^2$, $4ar/b^2=e^2$, $R+a-r=d$.
$2r/d=c$, $R-a+r=h$, $-2r/h=c_1$, $\sqrt{[1-e^2\sin^2\theta]}=D$.

$$\therefore\ V=\tfrac{32}{3}br^2\int_0^{\frac{1}{2}\pi}D\sin^2\theta\cos^2\theta d\theta+\tfrac{16}{3}abr\int_0^{\frac{1}{2}\pi}D\sin^2\theta d\theta+\tfrac{4}{3}b(a-r)^2\int_0^{\frac{1}{2}\pi}Dd\theta$$

$$+\frac{32ar^3}{3b}\int_0^{\frac{1}{2}\pi}\frac{\sin^6\theta d\theta}{D}+\frac{48ar^2(a-r)}{3b}\int_0^{\frac{1}{2}\pi}\frac{\sin^4\theta d\theta}{D}+\frac{8ar(4R^2-3b^2)}{3b}\int_0^{\frac{1}{2}\pi}\frac{\sin^2\theta d\theta}{D}$$

$$-\frac{4(a-r)(ab^2+2R^2r)}{3b}\int_0^{\frac{1}{2}\pi}\frac{d\theta}{D}-\frac{16R^3r^2}{3bd}\int_0^{\frac{1}{2}\pi}\frac{\sin^2\theta\cos^2\theta d\theta}{(1+c\sin^2\theta)D}$$

$$-\frac{16R^3r^2}{3bh}\int_0^{\frac{1}{2}\pi}\frac{\sin2\theta\cos^2\theta d\theta}{(1+c_1\sin^2\theta)D}.$$

$$\therefore\ V=\tfrac{4}{3}\Big[\frac{16br^2}{15e^4}(1-e^2+e^4)-\frac{4abr}{e^2}(1-2e^2)-\frac{8ar^3}{15be^6}(8+7e^2+8e^4)+b(a-r)^2$$

$$-\frac{24ar^2(a-r)}{3be^4}(1+e^2)-\frac{2ar(4R^2-3b^2)}{be^2}-\frac{4R^3r^2}{bcde^2}-\frac{4R^3r^2}{bc_1he^2}\Bigg]E(e,\ \tfrac{1}{2}\pi)$$

$$+\tfrac{4}{3}\Bigg[-\frac{8br^2}{15e^4}(2-3e^2+e^4)+\frac{4abr}{e^2}(1-e^2)+\frac{8ar^3}{15be^6}(8+3e^2+4e^4)$$

$$+\frac{12ar^2(a-r)}{3be^4}(2+e^2)+\frac{2ar(4R^2-3b^2)}{be^2}$$

$$-\frac{(a-r)(ab^2+2R^2r)}{b}-\frac{4R^3r^2}{bc^2de^2}(ce^2+e^2-c)-$$

$$\frac{4R^3r^2}{bc_1^2he^2}(c_1e^2+e^2-c_1)\Bigg]F(e,\ \tfrac{1}{2}\pi)+\frac{16R^3r^2}{3b}\Bigg[\frac{c+1}{dc^2}\Pi(e,\ c,\ \tfrac{1}{2}\pi)+\frac{c_1+1}{hc_1^2}\Pi(e,c_1,\tfrac{1}{2}\pi)\Bigg]$$

$$\therefore\ V=AE(e,\ \tfrac{1}{2}\pi)+BF(e,\ \tfrac{1}{2}\pi)+C\Pi(e,\ c,\ \tfrac{1}{2}\pi)+C_1\Pi(e,\ c_1,\ \tfrac{1}{2}\pi),\text{ suppose.}$$

Let $\dfrac{a^2+R^2-r^2}{2a}=x_1$; then from $a=R-r$ to $a=R+r$,

$$V_1=4\int_{x_1}^{R}\int_0^{\sqrt{(R^2-x^2)}}\sqrt{[R^2-x^2-y^2]}\,dxdy+4\int_{a-r}^{x_1}\int_0^{m}\sqrt{[R^2-x^2-y^2]}\,dxdy$$

$$=\pi\int_{x_1}^{R}\sqrt{[R^2-x^2]}\,dx+2\int_{a-r}^{x_1}mn\,dx+2\int_{a-r}^{x_1}(R^2-x^2)\sin^{-1}\left(\frac{m}{\sqrt{[R^2-x^2]}}\right)dx.$$

$$\therefore V_1=\tfrac{1}{3}\pi[2R^3-(a-r)(2R^2+b^2)]+AE_0^{\theta_1}(e,\ \theta)+BF_0^{\theta_1}(e,\ \theta)+C\Pi_0^{\theta_1}(e,\ c,\ \theta)$$

$$+C_1\Pi_0^{\theta_1}(e,c_1,\theta),\text{ where }\theta_1=\tfrac{1}{2}\cos^{-1}\left(\frac{a^2+r^2-R^2}{2a}\right).$$

$$\therefore\triangle=\frac{\displaystyle\int_0^{R-r}V\,da+\int_{R-r}^{R+r}V_1\,da}{\displaystyle\int_0^{R+r}da}.$$

A partial solution was given by *J. M. COLAW*.

MISCELLANEOUS.

83. Proposed by ALOIS F. KOVARIK, Instructor in Mathematics and Physics, Decorah Institute, Decorah, Iowa.

Segment's area of a circle whose radius is 5 inches is 28.56 square inches. Find the chord.